Rigorous bounding of position error estimates for aircraft surface movement

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Abstract

NextGen will require new navigation and surveillance capabilities to support safe and efficient surface operations based on tightly-coordinated 4D trajectories. In developing these new technologies, such as Automatic Dependent Surveillance-Broadcast (ADS-B) and the Ground Based Augmentation System (GBAS), it is essential to remember that all sensing technologies are prone to rare but potentially hazardous errors. Accordingly, the development of new navigation and surveillance technologies must be complemented by the development of rigorous integrity algorithms that allow pilots and controllers to determine when a sensor system should or should not be trusted.

This paper describes the application of a new state-prediction methodology to developing conservative position-error bounds for aircraft ground movement. These position-error bounds can be compared to operational limits in order to generate an alert if the risk of a large navigation error becomes unacceptably high. To demonstrate the conservatism of our approach, we have conducted a series of experiments in a lab setting using a surrogate (robotic) vehicle. These experiments indicate that our method, which we call biased-Gaussian prediction, generates a conservative position-error bound even when more conventional prediction methods do not.

Introduction

Accommodating significant increases in air traffic poses new safety challenges to the NextGen air traffic control system. In order to improve airport capacity and efficiency, aircraft will have to operate in much closer proximity to one another. New navigation and surveillance capabilities will be required to support decreased separation distances. Accordingly, the Joint Planning and Development Office (JPDO) vision for NextGen calls for increased reliance on GPS-based technologies, including ADS-B and GBAS. While more accurate positioning with GPS and other Global Navigation Satellite Systems (GNSSs) will allow decreased separation distances, these technologies, like all sensor technologies, are prone to errors. In the case of GNSS-based navigation, rare but hazardous errors may result from satellite clock faults or severe ionospheric storms. Rigorous integrity algorithms are needed to define error bounds to account for such events. In particular, integrity bounding methods are needed to support sensor fusion applications, in which GPS-based measurements may be supplemented by vehicle kinematic models and additional data, such as inertial measurements.

However, state-estimation algorithms that rigorously bound sensor error distributions are not yet well-developed. Much of the challenge is due to the fact that GPS measurements (and sensors in general) are prone to rare, off-nominal errors with heavy-tailed distributions. It is important to account for these non-Gaussian far-tail errors when developing safety-critical error bounds, such as error bounds used to verify aircraft separation. In the past, much research has focused on improving estimation accuracy by improving the methods used to represent the core of an error distribution [1-3]. By comparison, little work has been done to improve estimators’ representations of the far tails of error distributions.

In previous work, we proposed a theoretical approach to state prediction, which we call biased-Gaussian bounding [4]. This method assigns a rigorous bound to the far tails of error estimates for
nonlinear systems such as aircraft. The new method employs a Gaussian overbound with unsigned biases which bound higher-order (nonlinear) distortions of the error distribution that occur during state propagation. This solution has the benefit of being relatively computationally efficient while still ensuring integrity. To date, our development of this method has focused only on prediction (and not on correction, which is the second step in state estimation [5]).

This paper will discuss an experimental verification of our theoretical method and its application to airport surface-traffic management. To collect experimental data we use a surrogate vehicle whose kinematics are nonlinear and representative of aircraft surface movement. Previously, we have demonstrated our method’s performance only in simulation. The value of working with experiments, in addition to simulations, is that real-world issues may arise. In particular, error distributions encountered in the experiments, both for initial conditions and for sensor noise, were significantly different than the simplified Gaussian models we have used in prior simulations.

The remainder of this paper is organized as follows. The paper begins with a discussion of 4D trajectories for aircraft surface movement. Subsequently, we review our proposed strategy for integrity-assured state estimation and discuss its application to 4D trajectory-based operations. Then, we introduce our experimental test bed as a model for aircraft surface movement. Finally, we present the results of the experimental verification of our method and demonstrate that our algorithm rigorously bounds integrity risk in situations where other state-prediction algorithms, such as Extended Kalman Filtering and Particle Filtering, fail to do so.

### Kinematics of Surface-Movement Trajectories

In modeling 4D trajectories for surface movement, we assume that trajectories are constructed as a series of linked elements, each specified as a simple geometric curve. Systems for defining and communicating such elements have previously been proposed by other researchers [6]. For our purposes, we consider two types of elements that could be linked to form surface trajectories: straight line segments and curved circular arcs. The figure below illustrates an example of a trajectory composed of a curved arc between two straight line segments.

![Figure 1. Curved and straight elements describing surface movement trajectories.](image)

In this paper, we assume the aircraft’s state vector consists of three variables which describe its motion on a planar surface: its horizontal \((x,y)\) coordinates and its heading angle \((\theta)\). For the purposes of surface movement, we neglect aircraft altitude. The aircraft state vector \(x_k\) can change at each discrete time index \(k\).

\[
x_k = [x_k \ y_k \ \theta_k]^T
\]

(1)

It is assumed that the pilot sets the along-track velocity \((v)\) and angular velocity \((\omega)\) of the aircraft. A control vector can thus be defined as follows.

\[
u_k = [v_k \ \omega_k]^T
\]

(2)

In general, it is assumed that the actual control values may be perturbed by some external disturbance \(w_k\). The perturbed control vector is labeled \(\bar{u}_k\).
\[ \tilde{u}_k = [\tilde{v}_k, \tilde{\omega}_k]^T = u_k + w_k \]  

(3)

Based on these definitions for aircraft states and controls, it is straightforward to define kinematic relationships which can be used to propagate aircraft states in the prediction step of a state estimator. Such a kinematic model can be obtained by integrating the aircraft velocity over a time interval \( \Delta t \) [7].

On straight segments, the kinematic model is linear.

\[ x_k = x_{k-1} + g_{\text{straight}} (\theta_k, \tilde{u}_k) \]

(4)

\[ g_{\text{straight}} (\theta_k, \tilde{u}_k) = \begin{pmatrix} \tilde{v}_k \Delta t \cos \theta_{k-1} \\ \tilde{v}_k \Delta t \sin \theta_{k-1} \\ 0 \end{pmatrix} \]

On curved segments, the kinematic equations are nonlinear.

\[ x_k = x_{k-1} + g_{\text{curved}} (\theta_k, \tilde{u}_k) \]

\[ g_{\text{curved}} (\theta_k, \tilde{u}_k) = \begin{pmatrix} -\tilde{v}_k \sin \theta_{k-1} + \frac{\tilde{v}}{\omega_h} \sin (\theta_{k-1} + \tilde{\omega}_k \Delta t) \\ \tilde{v}_k \cos \theta_{k-1} - \frac{\tilde{v}}{\omega_h} \cos (\theta_{k-1} + \tilde{\omega}_k \Delta t) \\ \tilde{\omega}_k \Delta t \end{pmatrix} \]

(5)

The goal of the estimator is to propagate the state and a representation of its error through time, fusing all available sensor data with the above kinematic models to provide a position estimate that is as accurate as possible. In many state estimators, including the Extended Kalman Filter (EKF), the state error is modeled using a time-varying covariance matrix \( P_k \).

\[ P_k = E \left[ (x_k - \hat{x}_k) (x_k - \hat{x}_k)^T \right] \]  

(6)

The state-error model is critical for safety-of-life applications, because it can be used as a real-time metric to assess system risks. For instance, a conformance monitor might compare the state estimate to a desired 4D trajectory. The risk that any aircraft is nonconforming depends both on the measured deviation of the state estimate and on the error model for that estimate. Estimation errors are important to consider because a sensor error that masks a hazardous deviation might cause a conformance-monitor missed detection. The safety of relative positioning, for applications such aircraft separation, also requires an estimate of the size of state errors. Since loss of separation might result in a hazardous event (such as a runway incursion), it is essential that state-estimation provide a rigorous bound on the risks associated with large errors in the far-tails of the state-error distribution.

**Integrity Risk and Separation Minima**

Although existing estimation methods approximate the actual state-error distribution, they do not establish a rigorous bound on the probability of hazardously large errors. We propose a novel method for state-prediction, the first step in state-estimation. Our method is specifically designed to generate a rigorous bound on large errors.

The state-prediction approach we discuss here is inspired by research in validation of navigation integrity for GPS-based precision aircraft landing [8,9]. This earlier work did not incorporate vehicle motion models, and relied entirely on sensor measurements at each time step to assess integrity risk in real time. However, it is desirable to include motion models in estimating aircraft states to further enhance the accuracy and reliability of the state estimate.

For the purposes of assessing the risks of far-tail errors in real time, it is useful to define the concept of a protection level (PL) [10]. The PL is an error boundary that contains all but a very few large errors. The location of the PL contour is set such that the integrated probability that an error lies outside the PL does not exceed a specified integrity risk. Typically, the PL is computed as a box around an aircraft, as shown in the Figure 2. To assess whether or not it is safe to conduct a high precision operation, the PL is compared to an alert limit (AL) which indicates the largest error that is allowed. For a Category-I GBAS landing, for instance, the alert limit is 10 m and the
Methodology

This section provides an overview for a biased-Gaussian state prediction, a method which generates a rigorous error bound for state predictions. We previously introduced this method in [4]. The biased-Gaussian error bound is based on a conservative approximation of the actual error distribution as a Gaussian distribution subject to an unknown, unsigned bias. The method converts nonlinearities into the unsigned bias term, such that a PL based on the biased Gaussian is guaranteed to be conservative. The form of the biased-Gaussian state-prediction method resembles the form of an EKF prediction step with one additional equation to track the magnitude of the unsigned bias $\mu_k$.

$$\hat{x}_k = f(\hat{x}_{k-1}, u_k)$$  \hspace{1cm} (7)

$$\dot{P}_k = F_k P_{k-1} F_k^T + G_k Q G_k^T$$  \hspace{1cm} (8)

$$\mu_k = \mu_{k-1} + m_k$$  \hspace{1cm} (9)

At each time step, the total unsigned bias increases by an incremental amount $m_k$, the value of which depends on the aircraft’s trajectory. The equation below describes the form for the incremental bias $m_k$ when the process noise $w_k$ is small compared to the initial state-error distribution $P_0$.

$$m_k = \begin{bmatrix} R(\hat{\theta}_{k-1}) r_x \cdot \hat{x} + \| r_x \| \\ R(\hat{\theta}_{k-1}) r_y \cdot \hat{y} + \| r_y \| \\ 0 \end{bmatrix}$$  \hspace{1cm} (10)

where

$$R(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$r = \frac{v}{\omega} \begin{bmatrix} \sin \omega \Delta t \\ 1 - \cos \omega \Delta t \\ 0 \end{bmatrix}$$

In previous work [4] a Monte Carlo simulation was used to assess the integrity performance of the biased-Gaussian filter and compare it to the EKF and particle filter. These simulations indicated that our new method could reliably provide a bound for a curved error distribution, even when conventional state-estimation techniques did not.

Experimental Verification Procedure

In this paper, we present experimental results that provide further verification for our proposed approach. An important characteristic of these experiments, which impacts the integrity of state-prediction algorithms, was that that the error distributions were not single-peaked (like a conventional Gaussian) but multi-peaked.

For our experiments, a robotic vehicle called the iRobot Create was used as surrogate for aircraft. With this platform it is possible to emulate aircraft motion in a compact lab setting. In our experiments, the robot was commanded to turn in constant-velocity circles on the lab floor. Its position was tracked with an overhead camera. At the conclusion of each loop, the robot was programmed to return to its docking station to recharge, which allowed automated data collection over long periods of time. A total of 5,221 trials were performed.
To compare the integrity performance of the biased-Gaussian filter to the EKF and particle filter, we propagated error bounds using each method’s state-prediction step (with no sensor correction) using the known control commands $u$.

The error bound for the EKF predictor was based on the assumption of a Gaussian error distribution with covariance $P_k$. This covariance varied as a function of time, as described by (8). Error bounds were ellipsoidal surfaces computed by scaling the basic covariance ellipsoid by a factor $\rho$ (Some authors label this type of ellipsoid scale factor a Mahalanobis distance.) All points on the ellipsoid satisfy the following equation.

$$
(x_k - \hat{x}_k)^T P_k^{-1} (x_k - \hat{x}_k) = \rho^2
$$

Each error can be characterized in terms of its Mahalanobis distance $\rho$. Thus it is useful to characterize an error bound as an ellipse at a given distance $\rho$, which we label PL. Errors that lie outside the error bound (that is, errors for which $\rho > PL$) are integrity risks. Thus, the fraction of errors outside the bound must not exceed a specified integrity limit $R_{int}$. Assuming a three-dimensional Gaussian distribution, as is relevant for our model problem with three states, the relationship between the distance $\rho$ and the integrity limit $R_{int}$ is the following [4].

$$
R_{int} = \text{Probability}\{\rho > PL\} = \Phi(PL) - \int_{PL}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{PL^2}{2}\right) dV
$$

$$
= 1 - \text{erf}\left(\frac{PL}{\sqrt{2}}\right) + \sqrt{\frac{2}{\pi}} PL \exp\left(-\frac{PL^2}{2}\right)
$$

Thus, for any desired level of integrity risk $R_{int}$, the error bound for the EKF predictor was the ellipsoid given by (11) in terms of the predicted state $\hat{x}_k$, the predicted covariance $P_k$, and the ellipsoidal scale factor $PL$ computed by inverting (12).

Conceptually, a different method might be used to derive an error bound for the Particle-Filter (PF) predictor. In theory, if the PF were run with enough samples, one could obtain an exact representation of the error probability density function (PDF) [7]. For practical reasons, PFs are implemented with a finite number of particles, each modeling a small fraction of probability (equal to $NP^{-1}$, where NP is the total number of particles). In this light, an integrity bound might be defined as the convex hull of a set of particles whose total probability was at least as large as the complement of the integrity risk ($1 - R_{int}$). In practice, for safety of life applications, it is difficult to justify an error bound based on a set of randomized particle locations, unless the number of particles outside the bound is large ($R_{int} >> NP^{-1}$). Thus, alternative methods are needed to derive an error bound from a particle distribution as the integrity risk becomes increasingly small. In this work, we used $10^4$ particles, which were propagated through time without resampling. Thus, it would be difficult to justify an error bound based on the particles themselves beyond, say, an integrity risk probability of 0.01.

Given the limitations of computing an error bound directly from a particle distribution, we took an alternate approach to obtain an error bound for the PF predictor. Specifically, we computed the statistical mean and covariance of the particle distribution ($\hat{x}_k$ and $P_k$). Based on these, we computed an ellipsoidal error bound, in much the same manner as we computed an ellipsoidal error bound for the EKF predictor, using (11) and (12). This approach assigns the PL based on an assumption that the far tails of the error distribution are Gaussian. This Gaussian model is somewhat arbitrary; however, some form for the far tails of the error PDF must be assumed since the PF does not explicitly model them. The result of this assumption about the far tails implies that the EKF and PF bounds are both ellipsoidal (but not necessarily identical, since the mean and covariance values for each are generally different for nonlinear dynamic systems).

A third error bound was computed for the biased-Gaussian filter. This bound was computed as a distorted ellipsoid, where the basic covariance ellipse was shifted to account for the bias term of (9). In effect, errors were assumed to be Gaussian-distributed but subject to a spatially-varying bias. In
evaluating the Mahalanobis distance, a worst case bias (one which maximized distance) was assumed.

\[
(\|x_k - \hat{x}_k\| + \mu_k) P_k (\|x_k - \hat{x}_k\| + \mu_k) = \rho_{bg}^2
\]  

(13)

In essence, the biased-Gaussian method is based on the concept that the actual distribution could be transformed into a truly Gaussian distribution if each differential element of probability were shifted by an appropriate bias. Thus the probability of errors at any particular biased-Gaussian distance \(\rho_{bg}\) can be obtained using the Gaussian model of (12).

Because it is too computationally expensive to compute the evolution of the actual error PDF in real time, all of the above methods approximate the actual distribution. For safety-critical applications, however, it is essential that these approximations be conservative: the risk of an error outside a bound should be no more than that predicted by (12). Unlike the EKF and PF predictors, the biased-Gaussian predictor was specifically derived to approximate the actual error distribution in a conservative manner. In order to verify the biased-Gaussian methodology and to assess whether or not the EKF and PF might also be inherently conservative, our experiments were designed to evaluate the conservatism of all three error bounds.

The following approach was used to assess the performance of each error bound. First, we ran a large number of trials \((N = 5221\) trials). Second, we evaluated prediction error as a function of time for each trial. For each trial \(n\) and time \(k\), prediction error was characterized by its Mahalonobis distance \(\rho_n(k)\). Third, for each time \(k\), we computed what fraction of the trials produced a prediction error outside the error bound \((\rho_n(k) > PL)\). Since integrity risk is the probability of an error occurring outside a given error bound, this fraction is a good statistical representation of integrity risk. Thus, the condition for the error bound to be conservative is the following.

Conservative if

\[
\frac{1}{N} \sum_{n}^{N} [\rho_n > PL] < R_{int}
\]  

(14)

Note that the expression in square brackets is a binary number (with value 0 if false and 1 if true).

It is important to note that equation (14) is only valid if the number of simulation trials \(N\) is very large. Specifically, we considered \(N\) to be a sufficiently large number of trials if the number of points expected to lie outside the error bound was at least 10. As a consequence, we can only consider PL bounds out to a practical limit supported by the number of trials conducted. In our case, there were 5,221 trials so the comparison was only considered valid for integrity risk levels of \(R_{int}\) greater than about \(10^{-2}\) which, according to (12), corresponds to a PL value of 3.9.

Given this limitation, our experimental verification procedure was the following. For the biased-Gaussian filter, we computed error bounds at various levels of integrity risk \(R_{int}\), up to \(10^{-2}\). For each level of integrity risk, we tested for conservative bounds using (14). This procedure was repeated for the EKF and PF. The conservatism of each error bound was computed by defining an integrity ratio \(\gamma\). This ratio compares the observed integrity risk (numerator) to the predicted integrity risk (denominator):

\[
\gamma = \frac{1}{N} \sum_{n}^{N} [\rho_n > PL] \frac{1}{R_{int}}
\]  

(15)

For the system to be conservative, the observed integrity risk should be less than or equal to the predicted integrity risk. Thus, when the integrity ratio is less than one, the error bound is conservative. When the integrity ratio is greater than one, the error bound is not conservative.

**Initial Distribution**

In order to obtain the biased-Gaussian, EKF, and PF predictions, we must appropriately initialize the state estimate and state-error covariance matrix at time zero. This was done using a conventional statistical approach [12]. The values for initial mean
and covariance are given in Table 1, below. Subsequent testing indicated that the process noise noise was modeled as being approximately zero. A complete noise analysis is presented in [13].

Control commands for the robot (longitudinal and angular velocities) were assigned as constants so that trajectories would be circular. Values for the nominal control commands are given in Table 1 in units that correspond to the “ground truth” sensor (an overhead camera). The camera calibration (which showed a very low barrel distortion) gave a conversion factor of 28.5 pixels per inch.

As mentioned previously, an important characteristic of the initial distribution is its bimodal structure, shown in Figure 3. The bimodal shape of the distribution is apparently caused by the robot’s undocking routine. For each trial, the robot automatically mates with a docking station, which serves as an initialization point for the subsequent run. To undock, the robot travels in reverse along a straight line at one of two headings. These trajectories place the robot in one of two locations, as indicated by the lobes of the initial state distribution in Figure 3. Each lobe contains approximately equal populations. Of the 5,221 trials, 56% fell into the upper lobe, and 44% fell into the lower.

The appearance of this bimodal distribution underscores a problem encountered in practical implementation of conventional state estimators. Distinctly non-Gaussian initial distributions may be for the experiments was very small, so the process encountered, and nonlinear processes will tend to distort them further. Multi-modal distributions represent a problem for error bounding since the region near the mean is relatively devoid of sample points, creating a greater likelihood of more points outside the ellipsoidal bound when the bound is small. As a consequence, error bounds may fail to ensure integrity if they do not encircle all of the prominent peaks of a multi-modal error distribution.

Table 1. Statistical Characterization of the Experimental System

| Initial State Estimate | $\mu_0 = \begin{bmatrix} 169.64 \\
                           464.59 \\
                           -0.0173 
\end{bmatrix}$ pixels |
|-------------------------|-------------------------|
| Initial State-Error Covariance | $P_0 = \begin{bmatrix} 1.113$ pixels$^2$ & 9.727 pixels & -0.0401 pixels \\
                        9.727 pixels & 253.54 pixels$^2$ & -1.025 pixels \\
                        -0.0401 pixels & -1.025 pixels & 0.0045 pixels$^2$ \end{bmatrix}$ |
| Modeled Process Noise Covariance | $E[w_k w_k^T] = 0$ |
| Control Commands | $u_k = \begin{bmatrix} 62.7 \\
                        0.294 \end{bmatrix}$ pixels/s rad/s |

Figure 3. Initial distribution of experimental data, projected onto the x-y plane, showing a bimodal structure.
As compared to the EKF and PF predictors, the biased-Gaussian predictor required one additional initialization parameter: an initial bias $\mu_0$. This initial bias was set to a nonzero value to account for the separation of the two lobes.

$$
\mu_0 = \begin{bmatrix}
0 \\
14 \\
0
\end{bmatrix} \text{ pixels}
$$

(16)

As time progresses, the initial distribution is distorted by nonlinearities of the vehicle kinematics. The evolution of the distribution was measured using our experimental platform. In the experiments, the error distribution remained consistently bimodal, as illustrated by Figure 4. The figure offers snapshots of the experimentally measured error PDF at several moments in time: $k = \{1, 40, 80, 120, 160, 200, 240\}$.

(Note the sample interval for this system was approximately 1/15 sec.) At each moment a scatter plot of the x-y error distribution for that time is shown in reference to the nominal circular trajectory (dark red line).

**Figure 4. Overview of experimental distribution for time steps $k = \{1, 40, 80, 120, 160, 200, 240\}$.

*Experimental Results*

The experimental trials were used to check integrity ratio for each of the three methods of interest. We found that the bimodal distribution causes problems for conventional EKF and PF predictors. By comparison, the biased-Gaussian was conservative (in fact it was overly conservative) for all time steps, at all levels of integrity risk tested.

Integrity ratio is plotted below as a function of PL for several time steps ($k = \{1,5,10,15\}$). In each plot, the integrity ratio is shown in red for the EKF, green for the PF, and blue for the biased-Gaussian filter. Note that on the far right of the plots, integrity ratio values are noisy because there are not enough simulation points available to obtain a valid assessment of integrity risk. As described previously, our sample size limits analysis to PL = 3.9.

**Discussion**

Recall that an error bound is considered to be conservative when integrity ratio is less than one. From each plot in Figure 5, it is clear that the EKF and PF predictors are not conservative over a significant range of PL values. The problem is caused by the bimodal distribution. The bimodal error distribution has a very low density of trials near the mean, between the two peaks. For small values of PL, the bounding ellipsoid would be predicted to contain many trials; however, because of the bimodal characteristic of the actual error PDF, almost no trials fall in the bounding ellipsoid when the PL is small. In the experimental trials described by Figure 5, the error bounds for the EKF and PF predictor eventually encircled both peaks of the bimodal distribution (at PL values above 2). Both bounds were conservative beyond this point. For a more general multi-modal error PDF, however, the distribution peaks might be arbitrarily far apart, making it difficult to ensure integrity even for much larger values of PL.

By comparison, the ability to set the initial bias for the biased-Gaussian filter provides a mechanism to ensure integrity for all PL values, even in the presence of the bimodal error distribution for the
The experimental system. In effect, the biased-Gaussian method shifts the two peaks toward one another, through the unsigned bias, to make the initial distribution appear to be a single-peak Gaussian. As long as the initial bias is sufficiently large, conservatism is guaranteed for all values of PL and all time steps.

It is important to note, however, that the biased-Gaussian filter’s integrity ratio is significantly less than one. This indicates that the method is highly overconservative. Ideally, the integrity ratio would be everywhere less than one (indicating conservatism), but otherwise as close to one as possible (so that the error bounds are no larger than what is necessary). The overconservatism of the biased-Gaussian method results because the unsigned bias grows in an unbounded manner. In order to stabilize the growth of this bound, we will need to modify the state-estimator to include a measurement correction step that takes sensor data into account. The formulation of a state-corrector algorithm is a topic for future work.

The performance of all three methods might be further improved by using a multi-hypothesis approach [5]. For example, a multi-hypothesis EKF with two Gaussian kernels would represent the bimodal distribution observed in this experiment much more accurately than the single kernel EKF described above. Thus, it is possible a multi-hypothesis EKF would remain conservative for all PLs, even if the conventional EKF did not. On the other hand, a multi-hypothesis biased-Gaussian model would need a much smaller initial bias; consequently a multi-hypothesis biased-Gaussian would likely be much more accurate (and much less overly conservative) than the single-kernel implementation of the biased-Gaussian method described here. A performance analysis for multi-hypothesis methods is a second topic for future work.

**Conclusions**

This paper presented the application of a specialized state-estimation algorithm, called the biased-Gaussian algorithm, to an airport surface-traffic management scenario. More specifically, we focused on the prediction step of state-estimation and showed that the biased-Gaussian predictor ensures a safe error bound, in the sense that the bound contains all but a small fraction of errors. (This fraction is called the integrity risk).
Through experiments, which replaced aircraft with surrogate robotic platforms, we showed that the biased-Gaussian algorithm remains conservative in several situations in which other prediction algorithms (such as those used in an EKF or PF) do not ensure integrity. This failure to ensure integrity was primarily due to the bimodal characteristics of the experimental error distribution.

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