BIOGRAPHY

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ABSTRACT

This paper describes a new method for making angle of arrival measurements in the terahertz (THz) frequency domain. Preliminary work towards a novel device is presented. The proposed device uses a movable diffraction grating to mimic phased antenna arrays used at radio frequencies (RF). Phased arrays are not practical with current THz equipment because of the cost and relative infancy of the technology. The THz interferometer device proposed in this paper sweeps a diffraction grating in front of a single detector. The pattern of measurements can then be used to estimate the signal’s angle of arrival. Two different configurations are compared: a single-slit diffraction grating is used to mimic rotating antenna and radar systems, and a double-slit diffraction grating is used to mimic phased arrays. Because the double-slit pattern has multiple peaks, a wide field of view can be achieved with a short grating sweep. This minimizes the time spent scanning dead space, allowing continuous or near continuous tracking of multiple transmitters distributed over a wide field of view for position estimation and communications. To the author’s knowledge, this novel device is the first to use a movable diffraction grating to make angle of arrival measurements in the THz range.

INTRODUCTION AND MOTIVATION

Angle of arrival measurements are used in location estimation problems for everything from aircraft to automobiles to cell phones [1–5]. The precision of angle measurements is a key limiting factor to accuracy in these problems. When projected over a long baseline, even small angle measurement errors can result in large position estimate errors. As a result, very precise angle of arrival measurements are necessary in relative positioning applications. This paper focuses on one specific application, the navigation and control of military cargo aircraft flying in formation to perform precision airdrop; however, the methods developed here may be used in a number of other applications including civilian formation flight control and collision avoidance, autonomous vehicle navigation, and robotics.

The military regularly uses precision airdrops to quickly deliver supplies and personnel to remote locations, often inside hostile territory [6,7]. One recent example is the delivery of humanitarian aid to Yazidi civilians in Northern Iraq. During the summer of 2014, tens of thousands of members of this religious minority became trapped in a mountainous region near the border with Syria by the advance of Islamic State (IS) militants. In addition to the threat of violence posed by the militants, it was feared that many could die of hunger and dehydration without access to food and water. In response to this crisis, a group of C-17 and C-130 cargo aircraft from the United States airdropped hundreds of thousands of pounds of food and water to the civilians, averting the immediate threat and giving many the chance to evacuate [8,9].

When performing precision airdrops, military aircraft typically fly in formation to ensure both precise package delivery and mutual protection from adversaries [10]. Because wake turbulence from leading aircraft is dangerous to both the following aircraft and the parachutes [11], it is essential that aircraft maintain precise relative positions during airdrop operations [12]. This is achieved using Station Keeping Equipment (SKE), which provides precise relative position estimates.

Traditional SKE systems use radio frequency (RF) signals to measure the range and bearing angle between aircraft. These signals, however, can propagate long distances and are easily detected, potentially giving an adversary significant advance warning of a formation’s approach [6,10]. Alternatively, GPS can be used for relative positioning, but the vulnerability of GPS to jamming via radio-frequency interference (RFI) is a well-known weakness of GPS-based systems [13,14]. As a result, there is demand for new relative positioning systems that are both stealthy and resistant to jamming.
In previous work [15, 16], we proposed such a system for measuring the range and bearing between aircraft pairs using THz signals. The THz band lies between microwaves and infrared radiation on the electromagnetic spectrum at the intersection of electronic and optical frequencies [17, 18], as shown in Figure 1. This frequency band has historically been difficult to exploit [19, 20] and is often referred to as the THz gap. Recent advances have begun to close the gap [21, 22], making THz technology available for a wide range of applications, including communications and networking, astronomy, medical imaging, security scanning, and non-destructive evaluation of materials and coatings [23–25].

The unique propagation properties of THz radiation offer a number of advantages for the precision airdrop application. THz signals are attenuated by atmospheric gases, severely limiting their detectability beyond a given propagation envelope [18]. Importantly, this attenuation is highly altitude dependent. At aircraft cruising altitudes, the attenuation is low, and transmission distances of several kilometers can be readily achieved; whereas at low altitudes, the attenuation is orders of magnitude higher, dissipating the signal after it travels relatively short distances [26, 27]. By carefully selecting the carrier frequency and transmit power, it is possible to design a THz communication system that is capable of establishing links over a few kilometers at altitude, while only allowing links of a couple hundred meters near ground level. This means that aircraft at altitude could communicate freely, and the signals would be essentially undetectable to a ground-based observer. In addition, the high attenuation at low altitudes would make it nearly impossible to jam the system from the ground. As a result, a THz relative positioning system offers significant stealth and jamming resistance advantages over current technology for the precision airdrop application.

Our past work [15, 16, 28] established that range measurements could be made, and focused primarily on the more challenging measurement of angle. Various phased array approaches, similar to those used in RF applications, were examined for measuring the signal’s angle of arrival. The high frequency and relative infancy of THz equipment, however, present some unique challenges. First, the equipment is expensive, making it desirable to use as few detectors as possible. Second, currently available THz detector equipment is often large relative to the wavelength, with diameters of several wavelengths, making it impossible to place two detectors very close together. Finally, currently affordable detectors are not capable of tracking the phase of the carrier signal, making it impossible to compare the phases from multiple detectors in electronics as is common in RF phased arrays. This paper, therefore, investigates a novel method for measuring angle of arrival using a movable diffraction grating and a single detector.

To the author’s knowledge, this is the first example of a movable diffraction grating being used to make angle of arrival measurements on THz signals. There has been some recent work investigating THz frequency diffraction patterns [29] and diffraction gratings [30]. In addition, movable diffraction gratings have been used in other frequency bands to allow a single detector to scan an entire diffraction pattern [31]; however, these two elements have not previously been combined. The combination is useful because, as will be described, the diffraction grating acts in an analogous manner to an RF phased antenna array, inducing interference to find the signal’s angle of arrival. This approach is particularly well suited to THz signals because it allows the device to access carrier information without having to track the carrier phase and using a single detector.

**BRIEF DESIGN SUMMARY**

The proposed system is composed of two primary hardware components, a THz transmitter package and a THz interferometric receiver package, as shown in Figure 2. In this work, it is assumed that the geometry is two-dimensional, in other words that altitude differences are negligible, and position can be determined from range and angle measurements.

![Figure 2: THz receiver package makes range and angle measurements using the signal from the THz transmitter package.](image-url)
**THz Transmitter Package**
The transmitter package is composed solely of the baseline transmitter equipment, presented in a previous work [28]. It broadcasts the THz signals to the receiver package, and is assumed to be capable of modulating code and/or data onto the signal for communications and range measurement. The transmitter package is assumed to be composed of a frequency synthesizer, arbitrary waveform generator, amplifier multiplier chain, and a diagonal horn antenna.

**THz Interferometric Receiver Package**
The THz receiver package is the novel component of the system, and is assembled from three different elements: a diffraction grating, an actuator, and a detector, as shown in Figure 3.

**Diffraction Grating**
The diffraction grating is a thin opaque sheet with optically small, uniformly spaced openings that cause the THz signals to diffract as they pass through. The grating causes the signals to interfere with each other inside the receiver package, generating an interference pattern on the back wall. The pattern of interference depends on the size and spacing of the openings, and the signal’s angle of incidence on the grating, as described in the following section.

Conceptually, any number of slits could be used. This paper considers two grating configurations: one with a single slit, and one with two slits. It is possible to use a many-slit grating and that possibility will be briefly discussed, but is not the focus of this paper.

**Actuator**
An actuator is used to sweep the interference pattern. One logical way to achieve this is by moving the detector back and forth through the pattern, but in this case, it is simpler to move the grating because it is not composed of sensitive electronics and does not require electrical connections, as the detector does. The grating is assumed to sit on a track that allows the actuator to move it from right to left across the front of the receiver package.

**Detection Equipment**
The detector element, like the transmitter, is the baseline detector equipment from previous work [28]. A single THz detector is used to make measurements of the signal. It is composed of an antenna, an amplifier and detector, and a data acquisition system. The detector is stationary at the back of the interferometric receiver package, and takes measurements of the THz signal as the grating is swept in front of it.

**ANALOGY TO RF TECHNOLOGY**
Two equipment configurations are considered in this paper, one with a single-slit grating and one with a double-slit grating. Analogies can be drawn between these two equipment configurations and existing techniques used in the RF range, namely radar and phased antenna arrays.

In the single-slit configuration, the slit acts basically like a window, blocking the signal when it is not aligned between the transmitter and detector, and allowing it to pass through when it is aligned. As a result, the measured power is highest when the slit is aligned, and low when it is not. The angle, therefore, is found by actuating the grating and identifying the point of highest power. This is analogous to rotating antenna or radar systems used in RF applications. Both are scanning over a range of angles and identifying the point of highest power as the signal’s angle of arrival. The only difference is the field of view is limited to less than 180° for the single-slit grating.

In the double-slit configuration, an interference pattern is generated on the back wall of the receiver package due to slight differences in the distance the signal travels as it passes through different slits on its way to the detector. Again, an analogy can be drawn to RF technology, this time to phased antenna arrays. In these arrays, a different phase is observed by each antenna in the array due to slight differences in the distance travelled by the signal. The phases are then compared in electronics to determine the angle of arrival. The double-slit configuration is effectively doing the same thing, except the interference is happening in the air instead of in the electronics.
OBJECTIVE AND HYPOTHESIS

The goal of this paper is to identify a receiver package configuration that can achieve continuous or near continuous tracking of multiple transmitters distributed over a wide field of view in order to perform positioning and maintain communications. This is important for the precision air-drop application where timely, precise, and stealthy relative positioning and communications are necessary between multiple aircraft in a formation.

Two equipment configurations are presented and compared, the single-slit and double-slit configurations. It is hypothesized that the double-slit diffraction pattern will provide spatial aliasing that can be exploited to minimize the grating sweep range, allowing continuous or near continuous tracking of multiple targets with little impact on the power and accuracy.

DETAILED DIFFRACTION ANALYSIS

Diffraction is the flaring or spreading of waves as they pass by obstacles or through narrow openings. It is one of the wave-like properties of photons and its occurrence is described by the theory of quantum electrodynamics (QED) [32]. Diffraction gratings are engineered to induce diffraction, and can take a number of different forms. For this application, a simple grating is assumed, made from a thin sheet of opaque material with one or more thin vertical slits cut out. As photons pass through the slit(s), they diffract, and as long as the signal is coherent, their interference generates a diffraction pattern on the back wall of the receiver package. Detailed derivations of the patterns can be found in [33]. This section provides a quick summary and presents the key equations.

Single-Slit Diffraction

Figure 4 depicts a simple single-slit diffraction grating configuration, as viewed from above. Photons from the transmitter may take any one of a number of paths through the slit on their way to a point $O$ on the wall, passing through the top, middle or bottom of the opening. Because the path lengths are different for each of these paths, the photons will have different phases when they meet at the detector. This results in interference.

In QED, each path and corresponding phase is represented by a phasor, a unit vector that rotates over time. If the distance to the wall is much greater than the size of the opening ($D \gg a$), the various paths from the slit to the detector can be treated as essentially parallel. This drastically simplifies the phasor equations, and by integrating over the width of the slit, the single-slit power distribution is found to be

$$P(\theta) = P_m \sin^2 \alpha,$$

where $P$ is the power distribution, $P_m$ is the maximum or peak power, and the argument $\alpha$ is defined as

$$\alpha = \frac{\pi a}{\lambda} (\sin \theta + \sin \phi),$$

where $a$ is the width of the slit, $\lambda$ is the wavelength of the signal, $\theta$ is the angle from the center of the slit to the point on the wall, and $\phi$ is the signal’s angle of arrival [33]. The cardinal sine is defined here as $\text{sinc}(x) = \sin(x)/x$.

The shape of the single-slit diffraction pattern is shown in Figure 5 for a few different values of the ratio $a/\lambda$ (an angle of arrival $\phi = 0^\circ$ is assumed). The pattern is composed primarily of one central peak. Technically there are some small local maximums in the tails of the pattern, but they are so small relative to the central peak that they are not particularly useful and can be ignored. The width of the central peak varies from roughly $15^\circ$ to greater than $90^\circ$ in the figure with the slit width $a$. Narrow slits result in greater diffraction of the signal and a broader central peak. Wide slits result in less diffraction and a narrower central peak. The ratio $a/\lambda$, therefore, is the key parameter controlling the width of the peak.

It is important to note that the peak power $P_m$ is normalized in Figure 5 to highlight the difference in the width of the peaks. In reality, the peak power is also a function of the slit width $a$. Narrow slits allow less power through than wide ones, resulting in a reduction in the peak power. In addition, they spread the power out over a wider area. As a result, the peak power of a narrow slit configuration is significantly less than the peak power of a wide slit configuration.
Single-Slit Interference Patterns

Multi-Slit Diffraction
Figure 6 depicts a simple multi-slit diffraction grating configuration, as viewed from above. In the particular case shown there are three slits ($N = 3$), but the analysis below is generic and may be applied to a grating with an arbitrary number of slits.

Similar to the single-slit case, there are multiple paths a photon may take to get from the transmitter to a point $O$ on the wall. In addition to the infinite number of paths contained within any one individual slit, there are multiple slits that a photon may pass through, and each of these slits has a different path length associated with it, resulting in a phase shift and interference. Like before, different paths can be represented by phasors. For the multi-slit analysis, however, there are a finite number of slits, so a summation is used instead of an integral. The complete interference pattern thus can be written as

$$P(\theta) = P_m \sin^2 \alpha \left(1 - \frac{\sum_{n=1}^{N} p_n(\theta, \phi)}{N}\right)^2,$$  

where $N$ is the number of slits, $n$ is an index used to label the slits sequentially from 1 to $N$, and $p_n$ is the phasor associated with the $n$th slit. The phasor $p_n$ is defined as

$$p_n = \frac{\cos(2\pi \ell_n(\theta, \phi)/\lambda)}{\sin(2\pi \ell_n(\theta, \phi)/\lambda)},$$

where $\ell_n$ is the additional distance the photon needs to travel to pass though the $n$th slit. As an example, $\ell_3$ is shown in Figure 6. Here, the top slit has been labeled $n = 1$ with each subsequent slit iterating by one. The additional path length for the first slit thus is $\ell_1 = 0$. If it is again assumed that the distance to the wall is large relative to the diffraction grating area ($D >> (N - 1)d + a$), the paths can be assumed to be parallel, and the path difference $\ell_n$ is defined as

$$\ell_n = (n - 1) \frac{\pi d}{\lambda} (\sin \theta + \sin \phi),$$

where $d$ is the spacing between the slits.

Equation (3) is composed of two parts multiplied together. The first portion represents the contribution of single-slit diffraction, and is equivalent to Equation (1). The second portion represents the role of multi-slit diffraction and comes from the phasor analysis.

The single-slit pattern described by Equation (1) is a special case of Equation (3). When there is only one slit ($N = 1$), the summation in the second term is dropped and the term becomes the norm of a unit vector, which is one. This leaves only the first term.

The double-slit configuration ($N = 2$) is also a special case of Equation (3). In this case, the summation of phasors can be simplified [33], and the equation becomes

$$P(\theta) = P_m \sin^2 \alpha \cos^2 \beta,$$  

where the argument $\beta$ is defined as

$$\beta = \frac{\pi d}{\lambda} (\sin \theta + \sin \phi).$$

Figure 7 shows an example of a double-slit diffraction pattern broken down into its two components: the single-slit interference term and the double-slit interference term. The parameters $\alpha/\lambda$ and $d/\lambda$ have been arbitrarily set here to 1 and 2, respectively, to make the figure clear and easy to read. The single-slit term (the first term of Equation (6)) is shown as a green dash-dot line, the double-slit
The double-slit term is a cosine function, and this gives the pattern its multiple peaks. The parameter \(d/\lambda\) controls the spacing of the peaks. Large slit spacing \(d\) results in a pattern with many closely spaced peaks, and small slit spacing \(d\) results in a pattern with only a few peaks.

The single-slit term is a cardinal sine function, and this gives the pattern its bell shape. As described above, the width of the bell is controlled by the parameter \(a/\lambda\). Small values of the slit width \(a\) result in a wide bell, and large values of the slit width \(a\) result in a narrow bell.

The complete double-slit pattern is the two terms multiplied together. The key difference to notice between the double-slit and single-slit patterns is the presence of multiple peaks in the double-slit pattern. These peaks can serve as additional markers for identifying the angle of incidence \(\phi\). This is the advantage of the double-slit pattern, and, as described above, can be utilized to allow the device to maintain a large field of view while scanning only a small subset of angles.

It is important to note the role of the single-slit term (the bell curve term) in limiting the width of the pattern. The tail of the bell suppresses side peaks in the double-slit pattern, as shown in Figure 7. The width of the pattern, therefore, is controlled by the slit width \(a\), a fact that will be important in the discussion of field of view below.

**DESIGN CONSIDERATIONS**

Design decisions were made to achieve the objective: near continuous tracking of multiple signals from transmitters distributed over a wide field of view. Specifically, the slit width \(a\), slit spacing \(d\), and distance to the detector \(D\) in the interferometric receiver package were set based on considerations of the field of view, minimum grating sweep range, and power.

**Field of View and Slit Width \(a\)**

A wide field of view is necessary to observe multiple transmitters spread over a wide area, as in formation flight. The field of view of the receiver package is fundamentally limited to less than 180° by the geometry, and in practice will be further limited because very little power will reach the detector at high angles of incidence. As a result, a reasonable target field of view for this work is 90°.

In the single-slit configuration the field of view is determined by the sweep of the grating. Because the pattern is composed of one main peak, the angle can only be observed if the peak is observed. This means that the receiver package’s field of view is equivalent to the range of angles swept by the grating.

In the double-slit configuration, on the other hand, there are multiple peaks in the pattern. Because of the additional peaks, the angle can be observed even if the central peak lies outside of the range of angles swept by the grating, assuming the ambiguity can be resolved. As a result, the field of view in the double-slit case is related to the width of the pattern and the number of observable side peaks. A wide pattern with multiple side peaks allows a wide field of view, while a narrow pattern with only a few side peaks limits the field of view. The key design parameter is the width of the slits \(a_d\). As described above and shown in Figure 7, narrow slits result in more diffraction and a wide field of view, and wide slits result in minimal diffraction and a narrow field of view. The relationship between the slit width \(a_d\) and the field of view is given by

\[
a_d = \frac{\lambda}{\sin \theta_{fov,d}}, \tag{8}
\]

where \(\theta_{fov,d}\) is the location of the first dark fringe in the cardinal sine (sinc) term of the double-slit pattern from Equation (6).

Because the desired field of view is 90°, the angle \(\theta_{fov,d}\) is set to 70° in this case, which, accounting for the roll off towards the tail, comfortably provides a field of view of roughly ±45° to either side of center. From Equation (8), this gives a slit width of \(a_d \approx 1.06\lambda\). Given that the wavelength \(\lambda\) of THz signals is on the order of 1 mm, this yields a slit size that is practical and easy to manufacture.
Minimum Grating Sweep and Slit Spacing $d$

Continuous or near continuous tracking of the signal is necessary in the precision airdrop application to allow for uninterrupted positioning and communications. As a result, it is desirable to utilize a small grating sweep range to minimize the amount of time spent scanning dead space in the pattern.

In the single-slit configuration, the lone peak in the pattern can only be observed if it lies within the sweep area. This means that the grating sweep must cover the entire field of view to observe the signal’s angle of arrival. Because the desired field of view is 90°, this means that the grating sweep must span ±45° to either side of center.

In the double-slit configuration, on the other hand, the presence of multiple peaks in the pattern allows the angle to be determined even if the central peak lies outside the range of the grating sweep. As long as at least one peak is observed over the sweep, the signal’s angle of arrival can be estimated, assuming the ambiguity can be resolved, perhaps through initialization. The minimum range that the grating must sweep, therefore, depends on the spacing of peaks, with closely packed peaks allowing for a small grating sweep, and sparsely spaced peaks requiring a large grating sweep. An excessively small peak spacing, however, can make the ambiguity resolution challenging, so in this case the desired minimum grating sweep was set to one fifth the field of view, or 18°.

The peak spacing in the double-slit configuration, and therefore the minimum sweep range, is governed by the slit spacing $d_d$. A small slit spacing results in large peak spacing, and a large slit spacing results in small peak spacing. The relationship between the slit spacing $d_d$ and the peak spacing is given by

$$d_d = \frac{\lambda}{\sin \theta_{ps,d}},$$

where $\theta_{ps,d}$ is the location of the peak next to the central peak in the double-slit pattern.

To achieve the desired minimum grating sweep of 18°, the angle $\theta_{ps,d}$ is set to 15°, leaving 3° buffer to account for the non-linearity in Equation (6). From Equation (9), this gives a slit spacing of $d_d \approx 3.86\lambda$. Again, given that the wavelength $\lambda$ of THz signals is roughly 1 mm, this corresponds to an easily attainable slit spacing.

Power and Detector Distance $D$

After passing through the grating, the THz signals are diffracted and spread out as they travel toward the back wall of the receiver package. To minimize spreading power losses, it is desirable to position the detector as close to the grating as possible; however, as mentioned above, the interference pattern equations assume that the distance to the detector is much greater than the size of the diffracting area ($D >> (N-1)d+a$). To ensure this, the detector distance $D$ is set to

$$D = 10((N-1) * d + a).$$

In the double-slit case, based on the parameters defined above, Equation 10 gives a detector distance of $D_d = 49.3\lambda$. For THz signals this corresponds to a receiver package that is roughly 5 cm in size.

In the single-slit case, to make a fair comparison, the receiver package is assumed to be the same size as in the double-slit configuration ($D_d = D_s = D = 49.3\lambda$). To maximize the power then, the slit width $a_s$ is made as large as possible while still maintaining the assumption that the distance to the detector is much greater than the size of the diffracting area ($D >> a$). Using Equation (10) the maximum acceptable slit width is found to be $a_s \approx 4.93\lambda$.

Table 1 summarizes the design parameters selected to meet the specifications.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Slit Width $a$</th>
<th>Slit Spacing $d$</th>
<th>Detector Distance $D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double-slit</td>
<td>1.06\lambda</td>
<td>3.86\lambda</td>
<td>49.3\lambda</td>
</tr>
<tr>
<td>Single-slit</td>
<td>4.93\lambda</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Both</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SIMULATIONS OVERVIEW

Simulations were performed in MATLAB to compare the single-slit and double-slit diffraction grating configurations and determine whether either can achieve the objective laid out in this paper. Ultimately, the goal is to extract the signal’s angle of arrival from measurements of the pattern. The following simulations explore the shapes of the measured patterns as a first step toward this and provide some preliminary insights into how this might be achieved. The simulations presented here do not consider noise, as they are only focused on the shapes of the patterns. A brief overview of the simulations is presented in this section. See the appendix for more detail.

Assumptions

It is assumed that the baseline transmitter and receiver hardware is the THz equipment from [28], the specifications of which are shown in Table 2. The transmitter has a carrier frequency $f$ of 300 GHz, corresponding to a wavelength $\lambda$ of 1 mm. It transmits at a power $P_T$ of 30 mW with a spreading angle $\phi_s$ of 2.5°(total beam angle 5°). The detector has a diameter $D_{det}$ of 5.6 mm and processing by the receiver electronics results in a loss coefficient $\rho_{pr}$ equal to $32/\pi^3$. The signal is sampled at
a frequency $f_s$ of 1 GHz, and the receiver electronics integrate the signal over 450,000 samples in the single-slit case and 2,250,000 samples in the double-slit case, as described below.

Table 2: Baseline THz hardware parameters from previous work

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency $f$</td>
<td>300 GHz</td>
</tr>
<tr>
<td>Carrier wavelength $\lambda$</td>
<td>1 mm</td>
</tr>
<tr>
<td>Transmitter power $P_T$</td>
<td>30 mW</td>
</tr>
<tr>
<td>Transmitter spreading angle $\phi_t$</td>
<td>2.5°</td>
</tr>
<tr>
<td>Detector diameter $D_{det}$</td>
<td>5.6 mm</td>
</tr>
<tr>
<td>Processing losses $\rho_{pr}$</td>
<td>32/\pi²</td>
</tr>
<tr>
<td>Measurement frequency $f_s$</td>
<td>1 GHz</td>
</tr>
<tr>
<td>Single-slit integration constant $K_s$</td>
<td>450,000</td>
</tr>
<tr>
<td>Double-slit integration constant $K_d$</td>
<td>2,250,000</td>
</tr>
</tbody>
</table>

Table 3 summarizes assumptions about the locations of the equipment and corresponding physical constants. It is assumed that the transmitter and receiver packages are both mounted on aircraft flying in formation at an altitude $z$ of 10 km and a separation distance $r$ of 1 km. The attenuation coefficient $\alpha_{atm}$ of the THz signal at that altitude is $3 \times 10^{-3}$ dB/km [26].

Table 3: Assumptions about equipment location

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude $z$</td>
<td>10 km</td>
</tr>
<tr>
<td>Measurement range $r$</td>
<td>1 km</td>
</tr>
<tr>
<td>Attenuation coefficient $\alpha_{atm}$</td>
<td>$3 \times 10^{-3}$ dB/km</td>
</tr>
</tbody>
</table>

Pattern Simulations

First, two simulations model the power distribution on the back wall of the receiver package for both the single-slit and double-slit case (without taking into account the effects of the detector). These come directly from Equation (1) and Equation (6). The first simulation shows the case where the angle of arrival $\phi$ is 0°, and the second simulation shows the case where the angle of arrival $\phi$ is 20°. The peak power $P_m$ is calculated by considering power losses due to signal spreading and attenuation losses, and the size of the slits.

Measurement Simulations

Next, detector measurements are simulated, which account for the size of the detector and movement of the grating. Again, the first simulation shows the case where the angle of arrival $\phi$ is 0° and the second simulation shows the case where the angle $\phi$ is 20°.

The size of the detector is accounted for by numerically integrating the power distribution across the width of the detector, effectively smoothing the pattern.

The movement of the grating is simulated numerically by averaging measurements over the distance traversed by the grating during one measurement period. In both the single-slit and double-slit cases, it is assumed that the grating moves at a constant linear speed and takes 0.09 s to complete one pass. In the single-slit case, the grating scans between $\pm 45°$ and it is assumed that the receiver integrates the signal over $K_s = 450,000$ samples, or 0.45 ms, resulting in a total of 200 measurements over the length of one sweep. In the double-slit case, the grating scans between $\pm 9°$, and so because the grating covers one fifth the area in the same amount of time, the integration time is quintupled to $K_d = 2,250,000$ samples, or 2.25 ms, resulting in a total of 40 measurements over the length of one sweep.

RESULTS

Results from the THz interferometer simulations comparing the single-slit and double-slit configurations relative to the objective are presented in this section.

Pattern for $\phi = 0°$

Figure 8 shows the interference pattern simulation results for the case where the angle of arrival $\phi$ is 0°. The power density incident on the back wall is plotted as a function of the internal angle $\theta$. Note that this is a power distribution and has units of Watts per degree $\theta$. The power density for the single-slit pattern, shown as a solid blue line, is composed of a single central peak, spanning roughly 11° to either side of center, with a high maximum power density. The double-slit power density pattern, shown as a red dashed line, on the other hand, is composed of five similar-sized peaks, separated laterally by roughly 15°, with low peak power.

![Interference Pattern Power Distribution](image)

Figure 8: Power distribution in the single-slit and double-slit diffraction patterns when $\phi = 0°$

Pattern for $\phi = 20°$

Figure 9 shows the interference pattern simulation results for the case where the angle of arrival $\phi$ is 20°. The power
density, measured in Watts per degree $\theta$, is again shown as a function of the internal angle $\theta$. The single-slit pattern is shown as a solid blue line and the double-slit pattern is shown as a dashed red line. The patterns here are almost identical to the patterns in Figure 8, except shifted by $20^\circ$ to the right. The other difference to note is the slight asymmetry of the patterns around the central peak. Both patterns appear slightly stretched on the right side, with larger peak separations than those on the left side. This is due to the nonlinear argument $\sin \theta$ that appears inside the terms of Equation (1) and Equation (6).

![Figure 9: Power distribution in the single-slit and double-slit diffraction patterns when $\phi = 20^\circ$](image)

**Measurements for $\phi = 0^\circ$**

Figure 10 shows the measured interference patterns when the signal’s angle of arrival $\phi$ is $0^\circ$. Note that the units here are Watts, not Watts per degree, because power measurements are plotted as opposed to a power distribution. The power measured by the detector is shown as a function of the grating’s angular position, which is equivalent to the internal angle $\theta$. The measurements of the single-slit pattern, shown as a solid blue line, span $\pm 45^\circ$, the range of angles swept by the grating in that case. The measurements of the double-slit pattern, shown as a dashed red line, span a smaller range between $\pm 9^\circ$, which is the range of angles swept in that case. The decrease in sweep range allows longer integration times, which effectively boost the power, making the two peaks similar in size. In the single-slit case, the peak power measurement is roughly 58% of what it would be if there were no grating, and in the double-slit case it is roughly 52%.

![Figure 10: Simulated measurements of the single-slit and double-slit diffraction patterns when $\phi = 0^\circ$](image)

**Measurements for $\phi = 20^\circ$**

Figure 11 shows the measured interference patterns for the case where the signal’s angle of arrival $\phi$ is $20^\circ$. Again, the power, measured in Watts, is shown as a function of the grating’s angular position. Also like the previous plot, the single-slit measurements, shown as a solid blue line, span $\pm 45^\circ$, while the double-slit measurements, shown as a red dashed line, span $\pm 9^\circ$. The main difference to note between this simulation and the one shown in Figure 10 is that the central peak in both cases as shifted to the right. In the single-slit case, the central peak is still visible, but in the double-slit case it has moved outside the window scanned by the grating. Despite this, a peak is still visible in the double-slit pattern, the first side peak. The single-slit peak is roughly 52% of what would be measured with no grating, and the double-slit peak is roughly 40%.

![Figure 11: Simulated measurements of the single-slit and double-slit diffraction patterns when $\phi = 0^\circ$](image)

**DISCUSSION**

A number of differences between the single-slit and double-slit configurations are evident in the results shown above, which suggest that the double-slit configuration is capable of providing continuous or near continuous tracking of multiple transmitters distributed over a wide field of view, as is necessary for the precision airdrop application.
Continuous Tracking
Continuous or near continuous signal tracking is important for high speed communications and uninterrupted precise positioning. For communications, long outages significantly increase the risk of missed data bits and to avoid this, messages must be repeated many times, significantly slowing communication speeds. For positioning, frequent extended outages can limit the integration and correlation times used to make accurate measurements and filter noise, resulting in imprecise measurements.

The simulations above show that the single-slit pattern has large regions of dead space, where the power is nearly zero. Because it has only one peak, the grating must scan the entire field of view to observe the angle of arrival. As a result, continuous tracking and a wide field of view cannot simultaneously be achieved in the single-slit case. Conceptually, a peak-following control algorithm could be implemented to keep the grating locked onto the peak, but this presents a problem for multiple access scenarios, as discussed below.

The double-slit pattern, on the other hand, is composed of multiple peaks and therefore has minimal dead space within the field of view. Because the peaks are repeated at multiple different angles (spatial aliasing), only a small region of the pattern needs to be scanned to maintain a large field of view, assuming that the ambiguity can be resolved, perhaps through initialization. From the measurement simulations in Figure 10 and Figure 11, it can be seen that this results in a large portion of the scan time being spent at or near peak power, allowing near continuous tracking of the signal. Furthermore, because of the closely packed peaks and the averaging effect of the detector width, even the valleys in the measured pattern are not compete dead zones; they have some power. As a result, fully continuous tracking may be possible depending on the power threshold of the system.

Multiple Access
The ability to simultaneously track signals from multiple transmitters at different locations is essential for multiple access communications and positioning. This is important for formation flight applications, where it is necessary to locate and communicate with a number of aircraft scattered across a wide field of view.

Simulations show that in the single-slit case, the entire field of view must be swept by the grating to identify the angle of arrival. In a multiple access scenario, this means that for any one signal, the grating will spend a significant amount of time scanning dead space, making continuous tracking impossible. If, as suggested above, the grating were to follow a single signal in order to maintain continuous tracking on it, it would necessarily reject signals coming from other directions, preventing multiple access.

As a result, the single-slit configuration cannot simultaneously achieve continuous signal tracking and a wide field of view for multiple access scenarios.

In contrast, the spatial aliasing of peaks in the double-slit pattern means that signals coming from different directions may have peaks that overlap. Simulations show that the signal can be nearly continuously tracked using a narrow grating sweep, even if the central peak in the pattern lies outside the range of the sweep. As a result, multiple signals can be simultaneously tracked over a wide field of view with minimal signal interruption in the double-slit configuration.

Power
In order to maximize the accuracy of measurements and ensure reliable communications, it is necessary to maximize the power of the received signal.

The pattern simulations above clearly show that the single-slit configuration provides significantly higher power densities than the double-slit configuration. The difference in power density between the two cases in Figure 8 and Figure 9 is two-fold: first, because the double-slit grating blocks more of the signal, and second, because it spreads the power over multiple peaks.

In the measurement simulations, however, the power difference is almost completely eliminated by the increased integration time in the double-slit configuration granted by the shorter sweep area, as shown in Figure 10 and Figure 11. Because the grating covers one fifth the number of points, it can integrate power five times longer, making the received signal powers almost equal in the single-slit and double-slit cases. The trade-off, though, is noise. In addition to increased signal power, the longer integration time also means increased noise power. This means that while the raw signal power is quintupled by the increased integration time, the signal-to-noise ratio is only improved by a factor of $\sqrt{5}$.

It should be noted that both the single-slit and double-slit gratings result in a roughly 50% decrease in the received power as compared to no grating at all. This is to be expected since both gratings block some portion of the signal headed towards the receiver and also cause it to diffract and spread out.

CONCLUSIONS
The THz double-slit interferometer provides a compelling option for relative positioning during precision airdrop operations. The THz signals offer stealth and robustness to jamming for operations over hostile territory, and the interferometer design presented in this paper allows angle measurements to be made on the THz signal using currently available and affordable equipment.
Simulations show that the double-slit configuration allows near continuous tracking of multiple transmitters distributed over a wide field of view, while the single-slit configuration can only achieve either continuous tracking or a wide field of view. In addition, because of the increased integration time in the double-slit configuration, the signal-to-noise ratio is improved, largely making up for the apparent difference in power between the two cases.

For the precision airdrop application, where high precision angle measurements are necessary and high data rate communications may be advantageous, the double-slit THz interferometer therefore is the better choice.

**FUTURE WORK**

The key next step in this work is to implement an algorithm for estimating the angle of arrival based on the simulated measurements. Noise models can then be implemented to predict the accuracy of angle measurements.

Another potential area for further investigation is multi-slit (\( N > 2 \)) gratings. Because of the increase in the number of slits, there is an increase in the power density at the peaks; however, the increased size of the grating means that the size of the receiver package must also be increased. Interestingly, some preliminary work has suggested that usable patterns may exist in the region \( 2((N-1)d+a < D < 4((N-1)d+a) \) for some multi-slit gratings. This could make it possible to get the power benefits of the multi-slit grating while minimizing the overall size of the device.

**APPENDIX: SIMULATION DETAILS**

The THz equipment is simulated in MATLAB using the parameters and specifications described in Table 1, Table 2, and Table 3. The simulation is performed for both the single-slit and double-slit case. A link budget is used to calculate both the power incident on each slit in the grating and the baseline power that would be incident on the detector if no grating were present. The power \( P_{in} \) incident is defined as

\[
P_{in} = P_{T} \times 10^{-r \alpha_{atm}/10} \times \frac{A_{in}}{2\pi r^2 (1 - \cos \phi_{t})} \times \cos \phi,
\]

where \( P_{T} \) is the transmitter power, \( r \) is the distance between the transmitter and the receiver, \( \alpha_{atm} \) is the atmospheric attenuation coefficient, \( A_{in} \) is the area of incidence, \( \phi_{t} \) is the transmitter beam spreading angle, and \( \phi \) is the signal’s angle of incidence.

For the baseline power \( P_{0} \) when no grating is present, the area \( A_{in} \) is set to the area of the detector. For the cases when a grating is present, it is assumed that the slits are very tall, such that any diffraction in the vertical direction is negligible, and the rays can be assumed to only diffract in the horizontal direction. As such, the area \( A_{in} \) is calculated by multiplying the width of the slit \( a \) by the height of the detector \( D_{det} \).

Next, the peak power \( P_{m} \) is calculated by dividing the total power through the slots by the integral of the normalized power distribution. The peak power is thus

\[
P_{m} = \frac{N P_{slit}}{\pi} \int_{-\pi/2}^{\pi/2} P_{norm}(\theta, \phi) d\theta,
\]

where \( N \) is the number of slits, \( P_{slit} \) is the power through each slit as calculated from Equation 11, \( P_{norm} \) is a normalized power distribution from Equation 3, and \( \theta \) is the internal angle. The integration is performed numerically in simulation. Note that the peak power \( P_{m} \) has units of Watts per radian \( \theta \), because it is power distribution.

Next, measurements are simulated. The equipment is assumed to take samples at a rate of 1 GHz; however, to reduce computing time, a reduction factor of \( R = 10^4 \) is applied, making the simulated sample rate 100 kHz. The position of the grating at each measurement epoch (every \( 10^{-5} \) s) is simulated by assuming that the grating moves at constant linear speed, covering the range of the sweep in 0.09 s. At each epoch a simulated sample is calculated by integrating the power distribution from Equation 3 over the diameter of the detector. The simulated power sample \( P_{samp} \) is

\[
P_{samp} = \int_{\theta_{det,left}}^{\theta_{det,right}} P(\theta, \phi) d\theta,
\]

where \( \theta_{det,left} \) is the angular position of the left edge of the detector and \( \theta_{det,right} \) is the angular position of the right edge. This accounts for the width of the detector.

The THz equipment performs an integration of the signal by summing over \( K_{s} = 450,000 \) samples, or 0.45 ms in the single-slit case and \( K_{d} = 2,250,000 \) samples or 2.25 ms in the double-slit case. Applying the reduction factor \( R \) these become 45 and 225, respectively. This integration is simulated by averaging the samples over the reduced summations. This produces the simulated measurement and accounts for the effects of the grating’s motion.

The resulting simulated measurements are then plotted.

**References**


